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Quantum effects in the interaction of the exciton with a leaky quasi-mode cavity

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Abstract. We have studied quantum effects in the interaction of the exciton with a leaky quasi-mode cavity field. When the exciton is initially prepared in a superposition state which exhibits holes in its photon-number distribution, whereas the cavity field initially is in the vacuum state, it is found that there exists an energy exchange between the exciton and the cavity field. The exciton and the cavity field may exhibit sub-Poissonian distributions and quadrature squeezings. It is shown that there does not exist a violation of the Cauchy-Schwartz inequality, which means that the correlation between the exciton and the cavity field is classical.

PACS. 42.50.Ct Quantum description of interaction of light and matter; related experiments – 42.50.Dv Nonclassical field states; squeezed, antibunched, and sub-Poissonian states; operational definitions of the phase of the field; phasemeasurements – 03.75.Fi Phase coherent atomic ensembles; quantum condensation phenomena

1 Introduction

The quantum coherence of superposition states is a basic principle governing the quantum world. The question why macroscopic superposition states are not observed has been raised by Schrödinger in his famous cat paradox [1,2]. Recent experiments demonstrated that the coherent superposition and its loosing process can be observed in the laboratory, at least in the mesoscopic domain. In a recent experiment [3], a superposition of two different coherent states for an ion oscillating in a harmonic potential was created as the Schrödinger cat. In another experiment [4], two coherent states of a cavity mode were also superposed coherently by the atoms passing the cavity with large detuning. This paper is devoted to studying the quantum effects in the interaction of the exciton with a leaky quasi-mode cavity when the exciton is initially in a superposition state and the cavity field in a vacuum state. The motivation to study this kind of interaction is that it is useful for investigating the possible practical realization of quantum information processes, such as quantum computing and quantum communications [5]. This paper is organized as follows: In Section 2, the model and its solution are briefly given. In Section 3, we discuss the energy exchange between the exciton and the cavity field. In Section 4, we investigate the sub-Poissonian distribution. In Section 5, quadrature squeezings of the exciton and the cavity field are discussed. The exciton-cavity correlation is investigated. At last, a conclusion is given.

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2 Model

We consider a two-dimensional lattice of identical two-level molecules (here called a quantum well) in a leaky Fabry-Perot cavity. The Hamiltonian for the quantum well and the cavity field under the rotating wave approximation is written as [6]

$$H = \hbar\Omega S_z + \hbar\sum\omega_j\hat{a}_j^+\hat{a}_j + \hbar\sum g_j(\hat{a}_j^+S_- + \hat{a}_jS_+) \quad (1)$$

with the collective operators

$$S_z = \sum_{n=1}^{N} s_z(n), S_\pm = \sum_{n=1}^{N} s_\pm(n) \quad (2)$$

where $s_z(n) = \frac{1}{2}(|e_n\rangle\langle e_n| - |g_n\rangle\langle g_n|), s_+(n) = |e_n\rangle\langle g_n|$ and $s_-(n) = |g_n\rangle\langle e_n|$ are quasi-spin operators of the nth molecule. Here $|e_n\rangle$ and $|g_n\rangle$ are the excited state and the ground state of nth molecule, respectively. Ω is the transition frequency of the isolated molecules. $\hat{a}_j^+(\hat{a}_j)$ are creation (annihilation) operators of the field modes which are labeled by continuous index j and the field frequency of each mode is denoted by ω_j. The coupling constant g_j between the molecules and the cavity field is determined by $g_j = \eta\Gamma[(\omega_j - \Omega)^2 + \Gamma^2]^{-\frac{1}{2}}$, where η depends on the atomic dipole and Γ is the decay rate of a quasi-mode of the cavity with a frequency Ω. Making a bosonic approximation $\hat{b} = \frac{S_-}{\sqrt{N}}$ and $\hat{b}^+ = \frac{S_+}{\sqrt{N}}$ with $[\hat{b}, \hat{b}^+] = 1$, the interaction between the cavity field and the quantum well occurs *via* excitons. Thus, equation (1) becomes

$$H = \hbar\Omega\hat{b}^+\hat{b} + \hbar\sum\omega_j\hat{a}_j^+\hat{a}_j + \hbar\sum g(\omega_j)(\hat{a}_j^+\hat{b} + \hat{b}^+\hat{a}_j) \quad (3)$$

where $g(\omega_j) = \sqrt{N}g_j$. The Heisenberg equations of motion for the operators of the field modes $\hat{a}_j(\hat{a}_j^+)$ and the excitons $\hat{b}^+(\hat{b})$ are given by

$$\frac{\mathrm{d}\hat{b}}{\mathrm{d}t} = -\mathrm{i}\Omega\hat{b} - \mathrm{i}\sum_j g(\omega_j)\hat{a}_j, \frac{\mathrm{d}\hat{a}_j}{\mathrm{d}t} = -\mathrm{i}\omega_j\hat{a}_j - \mathrm{i}g(\omega_j)\hat{b}. \quad (4)$$

Equation (4) can be solved as [6]

$$\hat{b}(t) = \left[u(t)\hat{b}(0) + \sum u_j(t)\hat{a}_j(0)\right]\mathrm{e}^{-\mathrm{i}\Omega t} \quad (5)$$

$$\hat{a}_j(t) = \mathrm{e}^{-\mathrm{i}\omega_j t}\hat{a}_j(0) + \upsilon_j(t)\hat{b}(0) + \sum \upsilon_{j,j'}(t)\hat{a}_{j'}(0) \quad (6)$$

where

$$u(t) = \left[\cos\left(\frac{\Theta}{2}t\right) + \frac{\Gamma}{\Theta}\sin\left(\frac{\Theta}{2}t\right)\mathrm{e}^{-\frac{\Gamma}{2}t}\right] \quad (7)$$

$$\upsilon_j(t) = -\mathrm{i}g(\omega_j)\mathrm{e}^{-\mathrm{i}\omega_j t}\int_0^t u(t')\mathrm{e}^{\mathrm{i}(\omega_j-\Omega)t'}\mathrm{d}t' \quad (8)$$

$$u_j(t) = L^{-1}\left(\frac{g(\omega_j)}{p + \mathrm{i}(\omega_j - \Omega)}\tilde{u}(p)\right) \quad (9)$$

$$\upsilon_{j,j'}(t) = -\mathrm{i}g(\omega_j)\mathrm{e}^{-\mathrm{i}\omega_j t}\int_0^t u_j(t')\mathrm{e}^{\mathrm{i}(\omega_j-\Omega)t'}\mathrm{d}t' \quad (10)$$

and $\Theta = \sqrt{4M\Gamma - \Gamma^2}$ with $M = \frac{N\eta^2 L'}{c}$, L' is the length of the cavity and c is the speed of the light in the vacuum. L^{-1} denotes the inverse Laplace transformation,

$$\tilde{u}(p) = \left[p + \tilde{K}(p)\right]^{-1}, \quad u(t) = L^{-1}\left[\tilde{u}(p)\right] \quad (11)$$

with $\tilde{K}(p)$ the Laplace transformation of the general memory kernel function $K(t-t') = M\Gamma\exp[-\Gamma|t-t'|]$ [6]. $\upsilon_j(t)$ is also written as

$$\upsilon_j(t) = -\mathrm{i}g(\omega_j)\left[\frac{1 - \mathrm{i}\frac{\Gamma}{\Theta}}{2}\right]\frac{\mathrm{e}^{\mathrm{i}(\frac{\Theta}{2}-\Omega)t-\frac{\Gamma}{2}t} - \mathrm{e}^{-\mathrm{i}\omega_j t}}{\mathrm{i}(\frac{\Theta}{2} + \omega_j - \Omega) - \frac{\Gamma}{2}} - \mathrm{i}g(\omega_j)\left[\frac{1 + \mathrm{i}\frac{\Gamma}{\Theta}}{2}\right]\frac{\mathrm{e}^{-\mathrm{i}(\frac{\Theta}{2}+\Omega)t-\frac{\Gamma}{2}t} - \mathrm{e}^{-\mathrm{i}\omega_j t}}{\mathrm{i}(\omega_j - \frac{\Theta}{2} - \Omega) - \frac{\Gamma}{2}}\cdot \quad (12)$$

3 Energy exchange between the exciton and the cavity field

Considering a superposition state denoted as $|\psi(\xi,\phi)\rangle$

$$|\psi(\xi,\phi)\rangle = \eta(\sqrt{\xi}|\alpha_1\rangle + \mathrm{e}^{\mathrm{i}\phi}\sqrt{1-\xi}|\alpha_2\rangle) \quad (13)$$

where η is a normalization constant

$$\eta = \left[1 + 2\sqrt{\xi(1-\xi)}\mathrm{Re}(\mathrm{e}^{\mathrm{i}\phi}\langle\alpha_1|\alpha_2\rangle)\right]^{-\frac{1}{2}}. \quad (14)$$

It is shown that state $|\psi(\xi,\phi)\rangle$ exhibits holes in its photon-number distribution [7]. Note that when $\xi \to 1(\xi \to 0)$, one has that $|\psi(\xi,\phi)\rangle \to |\alpha_1\rangle(|\psi(\xi,\phi)\rangle \to |\alpha_2\rangle)$, where $|\alpha_j\rangle(j = 1, 2)$ is a coherent state. Hence, the field state in equation (13) interpolates between two arbitrary coherent states $|\alpha_1\rangle$ and $|\alpha_2\rangle$. In reference [7], Baseia *et al.* assumed that $\alpha_j = r\mathrm{e}^{\mathrm{i}\theta_j}(j = 1, 2)$ and showed that by setting $\xi = \frac{1}{2}$ and $\phi + n\triangle\theta = (2m + 1)\pi \quad (m = 1, 2, 3, \cdots)$, a hole is burned at the component $|n\rangle = |N\rangle$ if choosing

$$\phi = (1 - N/N_0)\pi, \quad \triangle\theta = \theta_2 - \theta_1 = \pi/N_0 \quad (15)$$

where N_0 is an integer.

If we prepare a superposition state for the system in which the exciton initially is in the state $|\psi(\xi = \frac{1}{2}, \phi, \triangle\theta = \frac{\pi}{2})\rangle = A(|\alpha_1\rangle + \mathrm{e}^{\mathrm{i}\phi}|\alpha_2\rangle)$, and the cavity field is in the vacuum state $\Pi_j|0\rangle_j$ (zero temperature), the whole initial state for the exciton and the cavity field can be expressed as

$$|\psi(0)\rangle = A(|\alpha_1\rangle + \mathrm{e}^{\mathrm{i}\phi}|\alpha_2\rangle) \otimes \Pi_j|0\rangle_j \quad (16)$$

where A is a normalization constant. The numbers of photons and excitons in this system evolve in the following way

$$\langle\hat{N}_{a_j}\rangle = 2r^2A^2|\upsilon_j(t)|^2[1 + \mathrm{e}^{-r^2}\sin(r^2 - \phi)] \quad (17)$$

$$\langle\hat{N}_b\rangle = 2r^2A^2|u(t)|^2[1 + \mathrm{e}^{-r^2}\sin(r^2 - \phi)] \quad (18)$$

where $2A^2 = [1 + \mathrm{e}^{-r^2}\cos(r^2 - \phi)]^{-1}$. It is obvious that there exists energy exchange between the exciton and the cavity field in the time evolution due to the following term

$$\langle\hat{N}_b\rangle - \langle\hat{N}_{a_j}\rangle = 2r^2A^2(|u(t)|^2 - |\upsilon_j(t)|^2) \times [1 + \mathrm{e}^{-r^2}\sin(r^2 - \phi)]. \quad (19)$$

Accordingly, the variances in the time evolution are given by

$$\langle(\triangle\hat{N}_{a_j})^2\rangle = 2r^2A^2|\upsilon_j(t)|^2[1 + \mathrm{e}^{-r^2}\sin(r^2 - \phi)] + 2r^4A^2|\upsilon_j(t)|^4[1 - \mathrm{e}^{-r^2}\cos(r^2 - \phi)] - 4r^4A^4|\upsilon_j(t)|^4[1 + \mathrm{e}^{-r^2}\sin(r^2 - \phi)]^2 \quad (20)$$

$$\langle(\triangle\hat{N}_b)^2\rangle = 2r^2A^2|u(t)|^2 1 + \mathrm{e}^{-r^2}\sin(r^2 - \phi)] + 2r^4A^2|u(t)|^4[1 - \mathrm{e}^{-r^2}\cos(r^2 - \phi)] - 4r^4A^4|u(t)|^4[1 + \mathrm{e}^{-r^2}\sin(r^2 - \phi)]^2 \quad (21)$$

where we have used relation $[\hat{a}_j, \hat{a}_j^+] = 1$.

4 Sub-Poissonian distributions of the exciton and the cavity field

Sub-Poissonian photon statistics of light are one of the best known non-classical effects [8]. We here investigate the sub-Poissonian distributions of the exciton and the cavity field under consideration. Following Mandel [9], the Q parameters for the exciton and the cavity field are introduced by

$$Q_b = \frac{\langle(\triangle\hat{N}_b)^2\rangle}{\langle\hat{N}_b\rangle} - 1,\ Q_a = \frac{\langle(\triangle\hat{N}_{a_j})^2\rangle}{\langle\hat{N}_{a_j}\rangle} - 1 \tag{22}$$

Sub-Poissonian exciton (photon) statistics exist whenever $-1 \le Q_{b(a)} < 0$. When $Q_{a(b)} > 0$, the state is called super Poissonian while the state with $Q_{b(a)} = 0$ is called Poissonian. We now calculate $Q_{b(a)}$ when the system is initially in $|\psi(0)\rangle = A(|\alpha_1\rangle + \mathrm{e}^{\mathrm{i}\phi}|\alpha_2\rangle) \otimes \Pi_j|0\rangle_j$. It is easy to get that

$$Q_a = \frac{-r^2|\upsilon_j(t)|^2\mathrm{e}^{-r^2}[2\sin(r^2-\phi)+\mathrm{e}^{-r^2}]}{[1+\mathrm{e}^{-r^2}\sin(r^2-\phi)][1+\mathrm{e}^{-r^2}\cos(r^2-\phi)]} \tag{23}$$

$$Q_b = \frac{-r^2|u(t)|^2\mathrm{e}^{-r^2}[2\sin(r^2-\phi)+\mathrm{e}^{-r^2}]}{[1+\mathrm{e}^{-r^2}\sin(r^2-\phi)][1+\mathrm{e}^{-r^2}\cos(r^2-\phi)]}\,. \tag{24}$$

When satisfying the inequality $2\sin(r^2-\phi)+\mathrm{e}^{-r^2} > 0$, we find that $Q_a < 0$ and $Q_b < 0$ which indicate that the exciton and the cavity field exhibit sub-Poissonian distributions.

5 Quadrature squeezings of the exciton and the cavity field

The quadratures for the exciton and for the cavity field are defined by

$$\hat{X}_b = \frac{1}{2}(\hat{b}+\hat{b}^+),\quad \hat{Y}_b = \frac{1}{2\mathrm{i}}(\hat{b}-\hat{b}^+),$$
$$\hat{X}_a = \frac{1}{2}(\hat{a}_j+\hat{a}_j^+),\quad \hat{Y}_a = \frac{1}{2\mathrm{i}}(\hat{a}_j-\hat{a}_j^+). \tag{25}$$

For the cavity field the degree of the squeezing may be characterized by the squeezing parameters [10]

$$S_{1a} = 2\langle\hat{N}_{a_j}\rangle + 2\,\mathrm{Re}\langle\hat{a}_j^2\rangle - 4(\mathrm{Re}\langle\hat{a}_j\rangle)^2 \tag{26}$$
$$S_{2a} = 2\langle\hat{N}_{a_j}\rangle - 2\,\mathrm{Re}\langle\hat{a}_j^2\rangle - 4(\mathrm{Im}\langle\hat{a}_j\rangle)^2. \tag{27}$$

If S_{1a} or S_{2a} is in the range $(-1,0)$, we say the cavity field exhibits quadrature squeezing. There are similar formulas for the excitons. We easily obtain for the system $|\psi(0)\rangle = A(|\alpha_1\rangle + \mathrm{e}^{\mathrm{i}\phi}|\alpha_2\rangle) \otimes \prod_j |0\rangle_j$,

$$\begin{aligned} S_{1a} = {} & 4r^2A^2|\upsilon_j(t)|^2[1+\mathrm{e}^{-r^2}\sin(r^2-\phi)] \\ & - 4r^2|\upsilon_j(t)|^2A^2\mathrm{e}^{-r^2}\sin(2\tau+2\theta_1)\sin(r^2-\phi) \\ & - 16r^2|\upsilon_j(t)|^2A^4\cos^2\left(\theta_1+\frac{\pi}{4}+\tau\right) \\ & \times\left[\frac{\sqrt{2}}{2}+\mathrm{e}^{-r^2}\cos\left(r^2-\phi-\frac{\pi}{4}\right)\right]^2 \end{aligned} \tag{28}$$

$$\begin{aligned} S_{2a} = {} & 4r^2A^2|\upsilon_j(t)|^2[1+\mathrm{e}^{-r^2}\sin(r^2-\phi)] \\ & + 4r^2|\upsilon_j(t)|^2A^2\mathrm{e}^{-r^2}\sin(2\tau+2\theta_1)\sin(r^2-\phi) \\ & - 16r^2|\upsilon_j(t)|^2A^4\sin^2\left(\theta_1+\frac{\pi}{4}+\tau\right) \\ & \times\left[\frac{\sqrt{2}}{2}+\mathrm{e}^{-r^2}\cos\left(r^2-\phi-\frac{\pi}{4}\right)\right]^2 \end{aligned} \tag{29}$$

where we have set $\upsilon_j(t) = |\upsilon_j(t)|\mathrm{e}^{\mathrm{i}\tau}$. It is easy to find that if selecting $r^2 = \phi$ and $\theta_1 + \tau = \frac{\pi}{4}$, $S_{1a} > 0$ and $S_{2a} < 0$, which means that for the cavity field, the Y_a component exhibits quadrature squeezing, but the X_a component does not.

Similarly, the expressions of S_{1b} and S_{2b} for the exciton are given by

$$\begin{aligned} S_{1b} = {} & 4r^2A^2|u(t)|^2[1+\mathrm{e}^{-r^2}\sin(r^2-\phi)] \\ & - 4r^2A^2|u(t)|^2\mathrm{e}^{-r^2}\sin(2\phi_u-2\Omega t+2\theta_1)\sin(r^2-\phi) \\ & - 16r^2A^4|u(t)|^2\cos^2\left(\theta_1+\frac{\pi}{4}+\phi_u-\Omega t\right) \\ & \times\left[\frac{\sqrt{2}}{2}+\mathrm{e}^{-r^2}\cos\left(r^2-\phi-\frac{\pi}{4}\right)\right]^2 \end{aligned} \tag{30}$$

$$\begin{aligned} S_{2b} = {} & 4r^2A^2|u(t)|^2[1+\mathrm{e}^{-r^2}\sin(r^2-\phi)] \\ & + 4r^2A^2|u(t)|^2\mathrm{e}^{-r^2}\sin(2\phi_u-2\Omega t+2\theta_1)\sin(r^2-\phi) \\ & - 16r^2A^4|u(t)|^2\sin^2\left(\phi_u-\Omega t+\theta_1+\frac{\pi}{4}\right) \\ & \times\left[\frac{\sqrt{2}}{2}+\mathrm{e}^{-r^2}\cos\left(r^2-\phi-\frac{\pi}{4}\right)\right]^2 \end{aligned} \tag{31}$$

where ϕ_u is determined by $u(t) = |u(t)|\mathrm{e}^{\mathrm{i}\phi_u}$. If setting $r^2 = \phi$ and $\phi_u - \Omega t + \theta_1 = \frac{\pi}{4}$, we find that $S_{1b} > 0$ and $S_{2b} < 0$, which indicate that for the exciton, the Y_b component exhibits quadrature squeezing, but the X_b component does not.

According to the above discussions, we conclude that the exciton and the cavity field may exhibit quadrature squeezing if properly adjusting parameters of the exciton-cavity system.

6 Exciton-cavity correlation

Correlation between the exciton and the cavity field may be characterized by the second-order correlation function (SOCF)

$$Q_{ab} = g_{ab}^{(2)}(0) - 1, \quad g_{ab}^{(2)}(0) = \frac{\langle \hat{N}_{a_j} \hat{N}_b \rangle}{\langle \hat{N}_{a_j} \rangle \langle \hat{N}_b \rangle} \cdot \tag{32}$$

The function Q_{ab} vanishes for uncorrelated states; it is positive for correlated states and negative for anti-correlated states.

For a system consisting of two bosonic modes (say a and b), there is the Cauchy-Schwartz inequality (CSI) [11]

$$[g_{ab}^{(2)}(0)]^2 \leq g_a^{(2)}(0) g_b^{(2)}(0) \tag{33}$$

where $g_a^{(2)}(0)$ and $g_b^{(2)}(0)$ are the second-order zero time correlation functions which are related to Mandel's Q parameters by

$$g_a^{(2)}(0) = 1 + \frac{Q_a}{\langle \hat{N}_a \rangle}, \quad g_b^{(2)}(0) = 1 + \frac{Q_b}{\langle \hat{N}_b \rangle} \tag{34}$$

where Q_a and Q_b are defined by equation (22).

It is shown [12] that the violation of the CSI can be accompanied by violation of Bell's inequality. If equation (33) is violated, the correlation between the two modes is non-classical, which can be described by

$$I_0(t) = \frac{\left[g_a^{(2)}(0) g_b^{(2)}(0) \right]^{\frac{1}{2}}}{g_{ab}^{(2)}(0)} - 1 \tag{35}$$

which is negative if equation (33) is violated. It is easy to calculate that

$$\langle \hat{N}_{a_j} \hat{N}_b \rangle = 2A^2 |u(t)|^2 |\upsilon_j(t)|^2 r^4 [1 - \mathrm{e}^{-r^2} \cos(r^2 - \phi)]. \tag{36}$$

Substituting equations (17, 18, 36) into equation (32), we have

$$Q_{ab} = \frac{-\mathrm{e}^{-r^2} [\mathrm{e}^{-r^2} + 2\sin(r^2 - \phi)]}{[1 + \mathrm{e}^{-r^2} \sin(r^2 - \phi)]^2} \cdot \tag{37}$$

When satisfying the inequality $2\sin(r^2 - \phi) + \mathrm{e}^{-r^2} > 0$ (under this condition, the exciton and the cavity field may exhibit sub-Poissonian distributions, respectively), we find that $Q_{ab} < 0$, which means that the correlation between the exciton and the cavity field is anti-correlated. In order to check whether the correlation is non-classical, we need to calculate equation (35).

Noticing that

$$g_{ab}^{(2)}(0) = g_a^{(2)}(0) = g_b^{(2)}(0) = \frac{1 - \mathrm{e}^{-2r^2} \cos^2(r^2 - \phi)}{[1 + \mathrm{e}^{-r^2} \sin(r^2 - \phi)]^2} \cdot \tag{38}$$

It is easy to find that $I_0(t) = 0$, which means that the correlation between the exciton and the cavity field is classical and no violation of the CSI occurs.

7 Conclusions

In this paper, we have studied quantum effects in the interaction of the exciton with a leaky quasi-mode cavity field. When the exciton is initially prepared in a superposition state which exhibits holes in its photon-number distribution, whereas the cavity field initially is in the vacuum state, it is found that there exists a energy exchange between the exciton and the cavity field. The exciton and the cavity field may exhibit sub-Poissonian distribution and quadrature squeezing, respectively. It is shown that there does not exist a violation of the Cauchy-Schwartz inequality, which means that the correlation between the exciton and the cavity field is classical.

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